

Differentiation Technique - Trigonometry

www.mymathscloud.com

Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing, Differentiation Technique – Trigonometry

Paper: Part A-Calc / Series: 2002 / Difficulty: Very Hard / Question Number: 3

3. An object moves along the x -axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

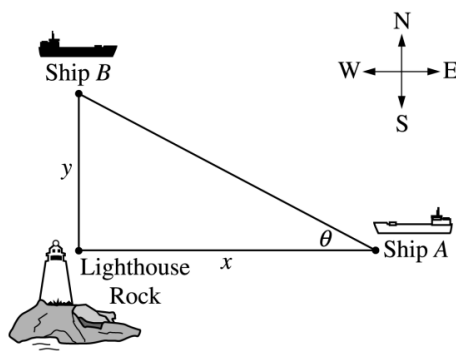
Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Related Rates, Implicit Differentiation, Modelling Situations, Differentiation Technique – Standard Functions, Differentiation Technique – Trigonometry, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Hard / Question Number: 6



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t , and let y be the distance between Ship B and Lighthouse Rock at time t , as shown in the figure above.
- Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.
 - Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.
 - Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 4$ km and $y = 3$ km.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 3

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 4

4. A particle moves along the x -axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.

- (a) Find the time t at which the particle is farthest to the left. Justify your answer.
- (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

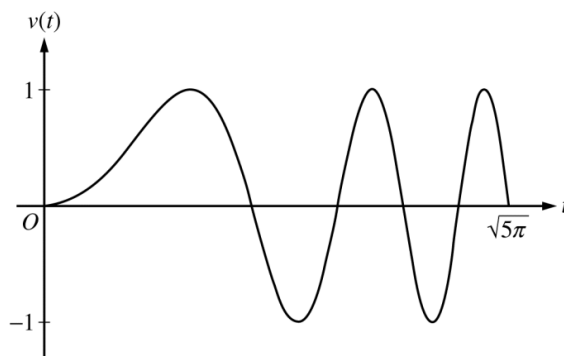
Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Differentiation Technique – Trigonometry, Global or Absolute Minima and Maxima

Paper: Part A-Calc / Series: 2007-Form-B / Difficulty: Hard / Question Number: 2



2. A particle moves along the x -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.
- (a) Find the acceleration of the particle at time $t = 3$.
 - (b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.
 - (c) Find the position of the particle at time $t = 3$.
 - (d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Tangents To Curves, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 4

4. The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2} \, dt$ and $g(x) = f(\sin x)$.

- (a) Find $f'(x)$ and $g'(x)$.
- (b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.
- (c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \leq x \leq \pi$. Justify your answer.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing, Differentiation Technique – Trigonometry, Average Value of a Function

Paper: Part B-Non-Calc / Series: 2010-Form-B / Difficulty: Somewhat Challenging / Question Number: 6

6. Two particles move along the x -axis. For $0 \leq t \leq 6$, the position of particle P at time t is given by

$$p(t) = 2\cos\left(\frac{\pi}{4}t\right), \text{ while the position of particle } R \text{ at time } t \text{ is given by } r(t) = t^3 - 6t^2 + 9t + 3.$$

- (a) For $0 \leq t \leq 6$, find all times t during which particle R is moving to the right.
 - (b) For $0 \leq t \leq 6$, find all times t during which the two particles travel in opposite directions.
 - (c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.
 - (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$.
-

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 7

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration, Differentiation

Subtopics: Continuities and Discontinuities, Average Value of a Function, Integration Technique – Exponentials, Integration Technique – Trigonometry, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Somewhat Challenging / Question Number: 6

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that f is continuous at $x = 0$.
 - (b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.
 - (c) Find the average value of f on the interval $[-1, 1]$.
-

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

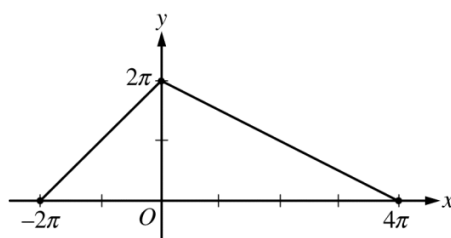
Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Differentiation Technique – Trigonometry, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Somewhat Challenging / Question Number: 6



Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and

let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

- (a) Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.
- (b) Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.
- (c) Let $h(x) = \int_0^{3x} g(t) dt$. Find $h'\left(-\frac{\pi}{3}\right)$.
-

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 9

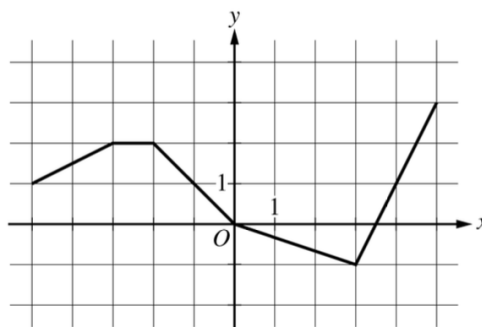
Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique – Chain Rule, Derivative Graphs, Differentiation Technique – Product Rule, Mean Value Theorem, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 6

x	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x .

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

- Find the slope of the line tangent to the graph of f at $x = \pi$.
- Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.
- Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.
- Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Rates of Change (Average), Tangents To Curves, Global or Absolute Minima and Maxima, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Product Rule, Differentiation Technique – Exponentials, Differentiation Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Somewhat Challenging / Question Number: 5

5. Let f be the function defined by $f(x) = e^x \cos x$.

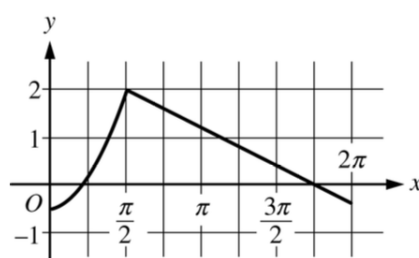
(a) Find the average rate of change of f on the interval $0 \leq x \leq \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \leq x \leq 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g' , the derivative of g , is shown

below. Find the value of $\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

Question 11

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Implicit Differentiation, Differentiation Technique – Trigonometry, Differentiation Technique – Product Rule, Tangents To Curves, Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Hard / Question Number: 5

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

- (a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.
- (b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.
- (c) For $0 \leq x \leq \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.
- (d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)